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BOARD OF ADMISSIONS AND RELATIONS WITH SCHOOLS (BOARS)
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Statement on Mathematics (Area C) Admissions Requirements
June 21, 2024

The University of California's Board of Admissions and Relations with Schools (BOARS) in fall 2023 convened a faculty Workgroup on Mathematics (Area C) Preparation to consider UC math admissions requirements in two stages. In Stage 1 the workgroup focused on the types of math courses that qualify as "advanced mathematics" for UC preparation (see Area C Workgroup Stage 1 Report). The workgroup's charge for Stage 2 was as follows:

1. Determine what coursework forms the most appropriate and necessary preparation for students to be successful at the University;
2. Consider whether UC Academic Senate regulations regarding math preparation should be updated; and
3. Engage with California State University (CSU) faculty and the Subcommittee on Mathematics Competencies convened by the Intersegmental Committee of the Academic Senates (ICAS) to align expectations for math preparation across the segments of California public higher education.

In addition, although not part of the original charge, the workgroup outlined the content of data science courses that would build on lower-level math coursework and effectively prepare students for quantitative coursework at the college level.

The Area C Workgroup's Stage 2 Report was submitted to and unanimously endorsed by BOARS.

Policy Guidance on Mathematics (Area C)

Based on the two Area C Workgroup (ACW) reports, BOARS' policy guidance on the math (Area C) admissions requirement, which will take effect starting in the 2025-26 school year, is as follows:

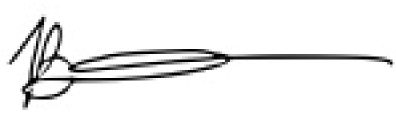
The ACW affirmed that the required three-course sequence of Algebra I–Geometry–Algebra II or the equivalent Math I–Math–II–Math III provides students with a foundation in mathematical fluency. Students who complete this sequence of courses (Category 1 courses) are eligible for full comprehensive review at UC and the CSU. As discussed in the ACW Stage 2 report, approximately 99.5% of UC applicants complete this course sequence.

The ACW described different types of Area C courses students could take as the recommended fourth year of mathematics. One group of courses (Category 2 courses) depends on mastery of the content in the required lower-level sequence and deepens this knowledge. The prime examples are calculus and precalculus courses. Category 2 courses offer the best preparation for students interested in Science, Technology, Engineering, and Mathematics (STEM) majors. Another group of courses

(Category 3 courses), such as AP Statistics, offers higher-level coursework in areas different from the required lower-level sequence and builds on students' prior mathematical understanding. Category 3 courses can provide effective preparation for students interested in quantitative social sciences. The ACW considers that together these two categories of courses (Category 2 and Category 3) best prepare students for a wide variety of college majors. A final group (Category 4 courses) includes other courses that do not fall into Categories 1-3, such as courses in quantitative reasoning, including data literacy. These courses will continue broadening students' interest and confidence in math and may be appropriate for students very likely to major in the arts and humanities. New labels for all categories of approved Area C courses will be developed and announced in the future by UC Undergraduate Admissions (see initial announcement in the June Counselors & Advisers Bulletin).

The ACW discussed how data science courses could be enhanced to offer better preparation for students interested in related fields at the University. The ACW strongly believes that data science is an area of growing importance and welcomes innovations in teaching data science at the high school level. In an appendix to their report, the ACW describes characteristics of potential data science courses that could build on mathematical concepts from the lower-level math sequence that would effectively prepare students for college-level coursework in data science.

A key point in the report is that **there are no recommended changes to UC Academic Senate regulations.** Rather, the ACW underscored the importance of the content of the required three-course sequence for all UC applicants and provided further guidance on the qualities of the recommended fourth math course that best prepares students depending on their educational goals. The workgroup urged high schools to offer a variety of fourth-year courses to meet student needs, including courses that deepen mathematical knowledge beyond the required sequence, to reduce barriers to entry into STEM fields.



Barbara Knowlton
BOARS Chair

UC Board of Admissions and Relations with Schools (BOARS)
Workgroup on Mathematics (Area C) Preparation, 2023-2024

Stage 2 Report
17 June 2024

Introduction

The Area C Workgroup convened by BOARS in Fall 2023 includes faculty in statistics, computer science, and mathematics. Our 12 members are drawn from the UC campuses, and, since January 2024, from the California State University (CSU) system as well. Some members, including the chair, have been responsible for the development and direction of undergraduate data science programs at our campuses.

As instructed, we have worked in close collaboration with the Subcommittee on Mathematical Competencies convened by the Intersegmental Committee of the Academic Senate (ICAS). Three members of that Subcommittee are also members of our Workgroup. In addition, we have worked with UC Undergraduate Admissions as well as with UC Transfer and Articulation.

Throughout, our overall goal has been to ensure that mathematics preparation standards enable access to UC for the broadest possible range of students, while also ensuring that all students arrive at UC adequately prepared to successfully pursue their academic goals.

Our charge had two stages. In Stage 1 we examined current UC admissions standards in mathematics. Our Stage 1 Report, completed in December 2023 and endorsed unanimously by BOARS, has four main components:

- We listed the coursework required for the essential mathematical foundation specified in Senate Regulation (SR) 424 A.3.c. This foundation includes “advanced algebra” which corresponds to the standard high school course Algebra II (or Math III).
- We determined that other courses cannot validate (substitute for, per SR 428) the “advanced algebra” requirement, unless they rely upon the overwhelming majority of content covered in Algebra II / Math III.
- We noted that there are no UC or California State standards in data science. We examined three specific high school curricula in this field and determined that they do not validate the required coursework as described above.
- We considered UC’s recommendation that students take a fourth year of mathematics in high school. We determined that this is a recommendation that

students build upon the required lower-level coursework, further developing their mathematics proficiency in preparation for study at UC. We also concluded that the three data science curricula we examined do not meet this criterion.

For Stage 2 we were charged with examining “what mathematics coursework forms the most appropriate and necessary preparation for students to be successful at the University” and determining whether the Senate Regulations on mathematics preparation need to be updated accordingly.

We would like to be clear at the outset that we are not proposing changes to Senate Regulations.

This Stage 2 Report has four main components:

- The Required Coursework. We provide data on the mathematical background of UC applicants, as well as the broad rationale for the required coursework. We examine the specific Algebra II content necessary for taking subsequent mathematics and statistics courses.
- The Recommended Fourth Year of Mathematics. We amplify our Stage 1 determination and provide a breakdown of the categories of courses in Area C.
- Domain-Specific High School Mathematics Preparation. We provide data on UC students’ majors, and recommend high school mathematics pathways for students depending on the UC majors they are considering.
- Data Science in the Fourth Year. We describe an overarching principle for creating data science content that is appropriate for a fourth year mathematics course that builds on the required lower-level courses.

In the Conclusion section we include some suggestions for reviewing and reinvigorating mathematics education in California at the high school level.

The Required Coursework

The Stage 2 charge asks us to determine “*what mathematics coursework forms the most appropriate and necessary preparation for students to be successful at the University.*” We define success as the ability of students to effectively pursue their evolving academic interests at UC, without facing excessive barriers because of choices made during high school.

UC-bound students have a diverse range of interests and intended majors. These majors have a wide range of mathematics prerequisites, and it would not be appropriate for UC to insist that all incoming students be equally prepared for all majors.

At the same time, part of the mission of UC is to foster a broadly educated and innovative Californian populace. Fundamental mathematical fluency is necessary for leadership in a world that is increasingly reliant on quantification and transformative technology.

As the Stage 1 Report states, “Regulation 424 A.3.c specifies that an essential mathematical foundation for study at UC includes elementary algebra, geometry, and advanced algebra. The Workgroup determined that these correspond to the standard high school course sequence Algebra I -> Geometry -> Algebra II (or the equivalent Math I -> Math II -> Math III).” For brevity, this report refers to these sequences as “the lower-level sequence.”

This lower-level sequence is needed for acquiring basic mathematical fluency. Understanding functions, rates, curves, cyclical patterns, maxima and minima, randomness, and so on, helps make sense of the economy, business, public and personal health, environmental changes, algorithms and their biases, patterns in language, and much more. It is also a necessary step toward understanding calculus, which is an indispensable tool in numerous fields including machine learning and data science.

Completion of the lower-level sequence satisfies the Area C requirement for full consideration for admission to UC. The requirement is simple and clear, and virtually all applicants from California satisfy it. For example, among the 132,413 California residents who applied for first-year UC admission in 2023, only 670 (0.5 of 1%) did not satisfy it.

Figure 1 summarizes the mathematics preparation of the California residents who applied for first year admission to UC in 2023.

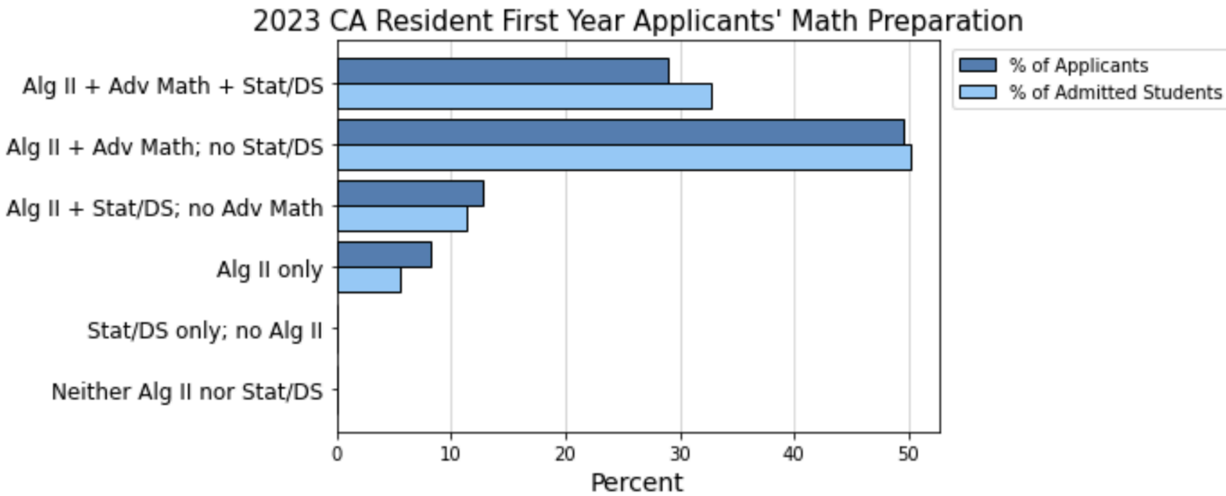


Figure 1: Highest level of Math preparation of CA resident UC first year applicants in 2023. Source: UC Undergraduate Admissions.

- Alg II stands for Algebra II.
- Adv Math stands for "Advanced Math" and includes precalculus, calculus (or higher), trigonometry, real analysis and other courses taken after the lower-level sequence, not including computer science, statistics, and data science.
- Stat/DS stands for "Statistics or Data Science". The majority of classes in this category are in statistics.
- The bars corresponding to the bottom two categories are too small to be visible.

Maintaining the requirement of the lower-level sequence will have no appreciable effect on the UC applicant pool. Figure 1 shows that the vast majority of UC applicants complete far more than this minimum requirement. For example, almost 80% of 2023 applicants from California (and over 80% of those who were admitted) took precalculus or even more advanced math in high school.

It is important to note that Algebra II or equivalent courses are not taught at UC and are limited in their availability at CSU and California Community Colleges. As a result, it is difficult for students lacking this knowledge to acquire it after entering higher education.

UC and CSU share the same minimum requirements for admission. This is critical for equity and opportunity: no high school student has to make an early choice between being UC-bound or CSU-bound. There has been concern at both UC and CSU about students arriving underprepared in mathematics, and CSU faculty have indicated strong support for requiring the lower-level sequence.

As charged for Stage 2, we also addressed the question, "*[W]hat do college-prep courses that address the content areas specified in Senate Regulation 424 A.3.c need to cover?*". In California, the content of the required lower-level sequence is defined by the California Common Core State Standards for Mathematics (CA CCSSM). It is not our role to propose changes to those State standards.

Nonetheless, an important consideration in deciding what the lower-level sequence "needs" to cover is whether it contains the specific material necessary for subsequent studies. We assessed which specific topics in the CA CCSSM Algebra II curriculum cover prerequisite content for subsequent mathematics classes commonly taken by UC applicants, specifically precalculus and statistics. This subset of Algebra II content is provided in Appendix A.

The Recommended Fourth Year of Mathematics

Beyond the required courses, SR 424 A.3.c recommends that students take a fourth year of mathematics in high school. As apparent in Figure 1, over 90% of UC first year applicants from California follow this recommendation.

Moreover, UC admission is competitive, and UC urges applicants to take the most rigorous courses available to them. Criteria for how applications are reviewed include the "[n]umber of, content of and performance in all A-G subject areas beyond the minimum requirements," and the "[q]uality of [the applicant's] academic performance relative to the educational opportunities available in [their] high school."

In this section we provide the rationale for our Stage 1 determination that, "Overall, a broad range of courses may be suitable for the recommended 4th year—provided that they are Mathematics courses of a level of mathematical challenge appropriate for 12th grade students already familiar with the lower-level required Mathematics coursework."

UC courses expect current fluency, not just past acquaintance, with the content and skills acquired in the lower-level sequence. To develop and maintain fluency, high school students need to keep their "mathematical muscle" trained and in good shape. Otherwise they risk arriving at UC less prepared in mathematics than they would have been at the end of 11th grade.

The most effective way to retain and solidify the knowledge of the first three years of math is by putting it to good use in the fourth year. That is why we concluded in Stage 1 that courses recommended for the fourth year of mathematics study are those that build substantially upon the knowledge and skills acquired in the required lower-level course sequence. The ability to reason mathematically grows, not just by knowing an increasing number of results, but by developing familiarity with increasingly abstract mathematical concepts and logical reasoning.

This is consistent with UC's Area C guidelines, which say, "*Courses will also recognize the hierarchical structure of mathematics, and advanced courses should demonstrate growth in depth and complexity, both in mathematical maturity as well as in topical organization*."

Area C thus breaks down into four categories of courses:

1. The required lower-level mathematics sequence.
2. More advanced courses that validate a lower-level course by requiring mastery of the content of that course, as described in the Stage 1 Report.
3. Recommended fourth year mathematics courses, as described in this section, that do not validate courses in the lower-level sequence (that is, do not rely on the overwhelming majority of the content of a lower-level course).
4. Other courses, for example courses in quantitative reasoning including data literacy, that do not fall into Categories 1-3.

We are mindful of factors that can constrain students' mathematics options. Importantly, 4% of California high schools are presently unable to offer mathematics courses beyond the lower-level sequence. At all schools, some students might progress exceptionally rapidly through the lower-level sequence; some schools are presently unable to offer such students two or more years of increasingly advanced courses appropriate for their mathematical development.

Resolving these problems may be an appropriate area for California state investment. In the meanwhile it is desirable, where practical, for students at such schools to be permitted to continue their mathematical development elsewhere, for example at Community Colleges, or through the UC Scout program that offers plans at no cost to California public schools and their students, or by taking other suitable courses online.

High schools may find Category 4 courses to be beneficial for some students. For example, such courses might help some students gain (or regain) confidence in mathematics, and might be a way for them to remain college-bound, whether to CSU or UC. However, schools should not channel students into Category 4 courses in their later high school years simply by making no other mathematics options available.

It is worth emphasizing once again that only the required courses (Category 1) are necessary for an application to receive full consideration. UC's recommendation of a fourth year is therefore not a limitation on access. Moreover, math classes beyond those that are required are just one part of a much larger application. Holistic review processes, such as those carried out by UC Admissions, are based on the entirety of each student's application and the educational opportunities available to them.

Domain-Specific High School Mathematics Preparation

UC students' choices of majors reflect the growing quantification of their world. Figure 2, adapted from publicly available UC data, shows the percentages of UC students majoring in various categories of disciplines over the past two decades.

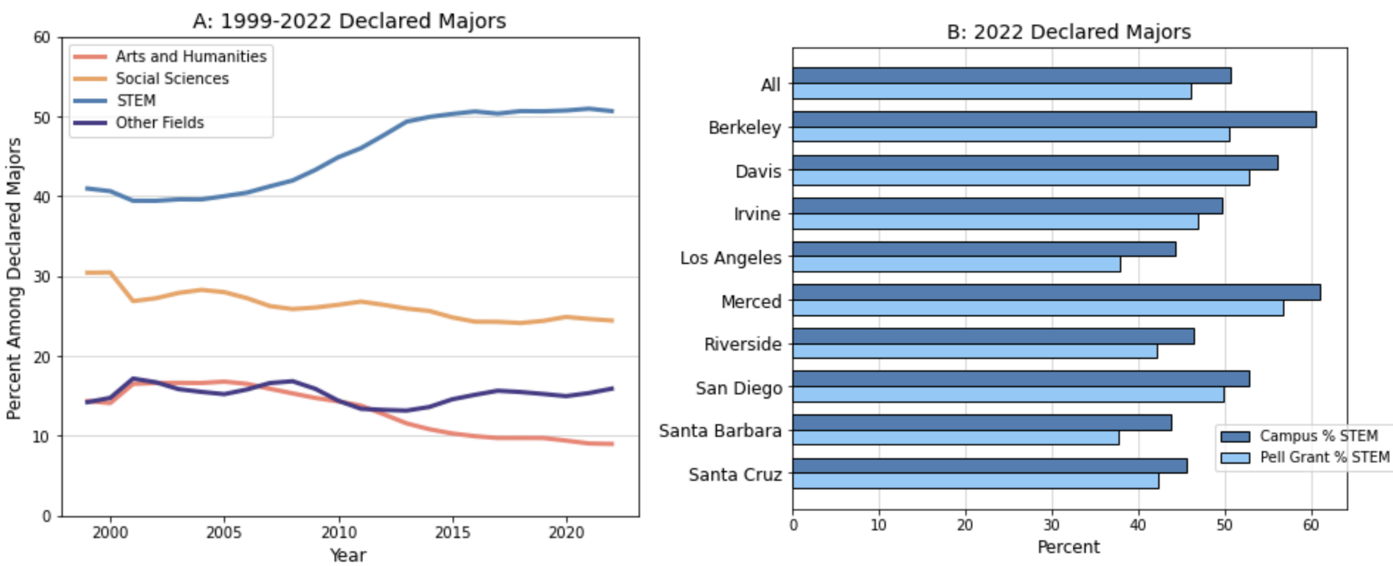


Figure 2: Major Domains of UC Students with Declared Majors. Source: Public UC data. Students whose major is Undeclared are not included. For students who declared more than one major, only the first major is included.

- Discipline is defined based on federal CIP codes as assigned to each major by UC campuses.
- The horizontal axis of Graph A represents academic years 1999-2000 to 2022-2023. Labels are abbreviated: for example, 2020 is short for the year 2020-2021.
- The data in Graph B are for 2022-2023 only. The two variables are the percent of STEM majors among all students with declared majors and the percent of STEM majors among Pell Grant recipients with declared majors.
- The San Francisco campus does not have an undergraduate program and is not represented.

STEM majors are popular among UC students including Pell Grant recipients. Figure 2A shows that in 2022-2023 at least 50% of all students with declared majors were in STEM fields. Figure 2B shows that this was also true of Pell Grant recipients with declared majors at four of the nine campuses. These include Merced which had the highest percentage of Pell Grant recipients among students with declared majors (62.3%) and also the highest percentage of STEM majors among those Pell Grant recipients (56.7%). By comparison, Berkeley had a far smaller percentage of Pell Grant recipients among declared majors (42.7%). But at Berkeley too, over half (50.5%) of these Pell Grant recipients were in STEM fields, as can be seen in Figure 2B.

The remainder of this section contains recommended high school mathematics pathways for first year applicants to UC, based on the broad domains of interest to UC students as evidenced by Figure 2, and on the requisite mathematics backgrounds for different majors.

STEM is not the only area that requires math. The economics major, which requires calculus and is classified under Social Sciences for almost all students in Figure 2, accounted for approximately 8% of students with declared majors in 2022-2023 (source: UCOP). Business-related majors at several UC campuses also require calculus.

For all UC STEM majors (including data science), as well as all other majors that require calculus, we recommend precalculus in high school, and we re-emphasize the importance of taking increasingly advanced math courses from one year to the next. The previous section includes our suggestions for high schools where such pathways are not available.

We would like to underscore a point that might seem counterintuitive: In order to major in data science at UC, it is more important for high school preparation to include precalculus rather than the existing high school data science courses.

Many UC students who are interested in STEM and other quantitative majors take precalculus and then calculus in high school. While these students move faster through their UC programs and can take advanced college courses early, it is not necessary to take calculus in high school. It is certainly possible to complete STEM majors on time if students take their first calculus class at UC; this is greatly facilitated if precalculus has been taken in high school.

Quantitative methods are increasingly important within the social sciences. Economics is a primary example. At present there is variability in mathematics requirements for other social sciences among campuses and fields. For example, the psychology major requires calculus at some UC campuses but not at others. Some B.A. degrees in cognitive science do not require calculus but others do, and all B.S. degrees in cognitive science require calculus. The sociology and political science majors do not require calculus, but quantitative and computational methods are used widely in those fields. We therefore recommend precalculus or Category 3 statistics courses as high school preparation for all these majors.

Majors within the arts and humanities typically do not involve substantial mathematics. These include history, languages, and the performing arts, to name just a few. Many UC applicants intent on these majors take precalculus or Category 3 statistics courses in high school to keep options open in college and also to take the most rigorous courses available to them. But some students with interests in the arts and humanities might prefer to take a Category 4 course in their fourth year of high school. Such a pathway may be quite reasonable for applicants who are sure they will not be adversely affected by the associated loss of academic flexibility after arriving in college.

The above recommendations are for high school students who have already identified future domains of interest. But many students simply do not know in high school which majors they might want to pursue in college. One of the great strengths of a UC education is the opportunity for students to make up their minds (and change their minds) about their intended major after they arrive at UC and learn more about academic disciplines and about themselves. We reiterate that the required lower-level sequence ensures that all students will have the math preparation to complete a broad range of majors at UC. All students will minimally have taken Algebra II or Math III. They will thus have the background to take precalculus or statistics at UC if necessary.

Data Science in the Fourth Year

In recent years there has been a burgeoning interest in data science education at the high school level. As charged in Stage 1, we examined some high school data science curricula. In the language of this report, we found that the three data science curricula most commonly used in California high schools fall into Category 4: neither validating a required lower-level mathematics course nor building sufficiently on the lower-level sequence to be a recommended fourth year mathematics course, but still teaching quantitative reasoning and data literacy. In this section we describe the main principle for developing data science content recommended for a fourth year of mathematics study, that is, content for Category 3 courses in data science.

As noted in the Stage 1 Report, there are no State standards at the high school level for data science. High school curricula labeled data science can therefore be markedly different from each other. Some most closely resemble classical applied statistics courses using menu-driven computational systems for data analysis.

Despite the differences, a common aspect of introductory high school data science curricula is the use of computer-based methods to almost entirely replace the mathematical formulation of the discipline. These curricula rely primarily on visualization and simulation to develop data literacy and to build intuition about data and sampling.

While this may be helpful and effective for some students, it can be a step backwards for students who have already completed the lower-level sequence.

Data science as a discipline has emerged from ever closer interactions among statistics (inference), computer science (algorithms), and mathematics (multivariable calculus, linear algebra, and optimization). By harnessing the content of the lower-level sequence, the students' knowledge of mathematics can be used to develop and refine their intuition about data, including inferential and algorithmic aspects as well as their tradeoffs.

The use of mathematics will help students see that results that seem plausible at first glance might in fact be incorrect. It will help reduce the doubts they might have about whether properties observed in a few simulations are true in greater generality. Most importantly, it will help demonstrate how the bedrock of data science is formed by mathematics along with computer science, probability and statistics, application domain knowledge, and expertise in the human context and ethics of data.

Data science courses suitable for Category 3 cannot shy away from mathematics. For example, it is not sufficient to use mathematical notation to define a quantity and then rely solely on computational black boxes to work with that quantity. Nor is it sufficient to present linear algebra as little more than a visual representation of data, without demonstrating its power to reveal patterns within large and complex data sets. If the mathematics that drives data science remains hidden, it not only deprives students of the opportunity to develop fluency in the math that they have previously learned, it also risks giving the impression that math is pointless or merely decorative.

The main principle in developing Category 3 data science courses must be to engage students substantively with the mathematics of the lower-level sequence while also making important points in data science. Appendix B contains six examples guided by this principle. They illustrate fundamentally important ideas in data science while also providing a workout in the mathematics of Algebra II or Math III. Specifically, they require students to understand and apply mathematical concepts such as functions and their inverses, the exponential function and the natural logarithm, polynomials, and inequalities.

Our examples are intended to illustrate how to weave mathematics together with aspects of data science. They are not prescriptive; we understand that curriculum developers will want to make their own choices of topics. As with all Category 3 courses, we hope that Category 3 data science curricula will aim to make lively and impactful use of the mathematics curricula of the first three years. We believe that it is possible for some of the existing introductory high school data science curricula to be modified, albeit with significant addition and deletion of content, to achieve this goal.

Conclusion

With Stage 2 at its end, we can now summarize our main conclusions from both Stages.

- We have not proposed changes to Senate Regulations 424 A.3.c and 428.
- The required mathematics preparation for UC is the standard high school course sequence Algebra I -> Geometry -> Algebra II (or the equivalent Math I -> Math II -> Math III), which we refer to as the lower-level sequence.
- For another course to validate a course in the lower-level sequence, it must require mastery of the content of that lower-level course.
- For a course to be recommended for a fourth year of mathematics study, it must build substantially on the content of the lower-level sequence.

Each year, well over 100,000 California high school students apply for first year admission to UC, and many more consider applying. Our conclusions affect not only the students but also the numerous people involved in their high school mathematics preparation, including families, teachers, schools, and school districts. As Figure 1 shows, our conclusions will not lead to any major changes for this large community, nor for UC admissions. The requirement is still the same lower-level sequence in algebra and geometry; virtually all applicants currently take these courses. As before, students are encouraged to take the most rigorous mathematics courses available to them; the vast majority of applicants already go substantially beyond the lower-level sequence.

We have provided some justification for the requirements and given some shape to the recommendations. We have considered the enormous diversity of potential UC applicants' academic backgrounds and interests, and we have offered some guidance on how students with vastly different interests can prepare themselves mathematically for success at UC. Our aim has been to help keep the doors of UC wide open while ensuring that all UC students—regardless of their high school—are equipped to succeed in the fields of their choice.

We have steadfastly kept our focus on carrying out our charge, which is to provide expert guidance to BOARS regarding the mathematics courses high school students should take in preparation for successful study at UC. It is well beyond our purview to perform a broad review of high school mathematics education in California. However, we are mindful that UC admissions standards are often used by non-UC bodies for other purposes. As a result, BOARS decisions can influence admissions to other colleges (including CSU), and the courses offered by high schools. We also recognize that many high school students find algebra courses, especially Algebra II, overfull of content and poorly motivated.

It is from this perspective that we would like to offer a few suggestions about high school mathematics education in California.

First, we would like to address a question put to us by the UC Regents at their March 2024 meeting: Is it possible to create a high school data science course that teaches Algebra II? We believe this to be a pertinent question in an era of growing interest in data science. It has long been the case that courses have utilized algebra to teach data science. We believe that it might be possible to turn the tables and utilize data science to teach algebra. The enterprise will require enormous creativity, content expertise, and pedagogical experience. It might result in a two-year course sequence that teaches both data science and Algebra II.

Not surprisingly, none of the existing high school data science curricula comes close to achieving this. Teaching algebra is not part of their goal. The project of creating the new course must start from scratch and will almost certainly take several years to complete. For positive outcomes in a shorter time, we suggest a smaller project that introduces motivating data science examples in the current curriculum of Algebra II. It is conceivable that UC and CSU faculty could contribute such examples.

Most importantly, we urge a renewed focus on how best to teach at the high school level the key mathematical skills required for success in college-level mathematics, especially calculus and abstract mathematical problem solving; see Appendix C. We ask for increased investment in research on what works to achieve this goal.

Sincerely, the Members of the Area C Workgroup,

(Chair) Ani Adhikari (Berkeley, Statistics)
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Mario Bañuelos (Fresno State, Mathematics)
Josh Berke (San Francisco, School of Medicine; BOARS)
Maribel Bueno Cachadina (Santa Barbara, Mathematics)
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Amit Sahai (Los Angeles, Computer Science)
Bruno Sansó (Santa Cruz, Statistics)
Katherine Stevenson (CSU Northridge, Mathematics)

Appendix A: Topics in Algebra II required in subsequent high school math

As noted in the section Required Coursework, the vast majority of UC applicants take a precalculus course in high school. Many take a fourth year statistics course such as AP Statistics, and many take both. Other students take precalculus or statistics as their first mathematics (or statistics, depending on the campus) course at UC. The required lower-level sequence must therefore prepare students adequately for these pathways.

We have determined that the topics below comprise the Algebra II content that covers the prerequisite knowledge for precalculus and statistics.

Our sources are the California Common Core State Standards for Algebra II (pages 77-84); the California Mathematics Framework's Precalculus chapter and Statistics and Probability chapter; the AP Precalculus curriculum, notably page 7; and the AP Statistics curriculum.

Prerequisite knowledge for precalculus consists of the following Algebra II topics.

- Proficiency with the skills and concepts related to linear and quadratic functions, including algebraic manipulation, graphing, solving equations, and solving inequalities
- Proficiency in manipulating algebraic expressions related to polynomial functions, including polynomial addition and multiplication, factoring quadratic trinomials, and using the quadratic formula
- Proficiency in using the Pythagorean Theorem and its applications, notably in trigonometry
- Proficiency in solving systems of equations in two and three variables
- Proficiency in manipulating simple rational expressions
- Familiarity with piecewise functions
- Familiarity with exponential functions and rules for exponents
- Familiarity with rules for logarithms and using them for solving exponential equations
- Proficiency in working with radicals (e.g., square roots, cube roots)
- Familiarity with complex numbers and their properties
- Familiarity with communicating and reasoning with graphical, numerical, analytical, and verbal representations of functions

Several items on the list above are also part of prerequisite knowledge for statistics courses appropriate for a fourth year of mathematics study. Chief among them are the modeling skills of reasoning with varied representations of functions, particularly algebraic and graphical representations of linear functions. Proficiency with exponential functions and rules for exponents is also essential, along with fluency in working with quadratic expressions and square roots. Underlying all of these skills is proficiency in manipulating algebraic expressions, such as polynomial addition and multiplication, and solving linear equations.

Additional statistics prerequisites consist of all of the Algebra II Statistics standards, namely sections S-ID, S-IC, and S-MD on pages 83-84 of the CA CCSSM.

Comparing the lists of topics covered in statistics in Algebra II and in statistics in a fourth year course appears to reveal a massive overlap. This kind of apparent overlap exists in the descriptions of introductory statistics courses at the college level as well. But it is largely an overlap in topic names, not in topic content. The more advanced course covers each topic in greater depth and with greater abstraction and mathematical complexity than the course that preceded it. For example, while Algebra II standards include the mean of a numerical data variable, the fourth year Statistics and Probability standards also include the closely related concept of the expectation of a random variable.

This feature of the two sets of standards is expressly described on page 3 of the fourth year Standards for Statistics and Probability: “In keeping with the California Common Core State Standards for Mathematics (CA CCSSM) theme that Mathematics instruction should strive for depth rather than breadth, teachers should view this course as an opportunity to delve deeper into those repeated Statistics and Probability standards while addressing new ones.”

Appendix B: Data Science Examples Appropriate for Fourth Year Mathematics

One of the principal conclusions of the Stage 1 report was that courses recommended for the fourth year of mathematics study are those that build substantially upon the knowledge and skills acquired in the required lower-level course sequence. In this report, the sections *The Recommended Fourth Year of Mathematics* and *Data Science in the Fourth Year* describe our rationale.

We now provide examples of data science content appropriate for a fourth year of mathematics study assuming that the student has completed the lower-level sequence.

The examples are guided by the following principles.

- They should make substantive use of the mathematics learned in the lower-level sequence, especially in the third year.
- The algebra involved should be elegant and illuminating, not heavy-handed or labor-intensive.
- The result for the student should be a deeper understanding of mathematics as well as data science.

Some mathematical details are provided to indicate the level. Prerequisite Algebra II content is referred to by labels for standards listed in the California Common Core State Standards in Mathematics (pages 78-84). The descriptions use technical language for a readership consisting of content developers. Text for students should use technical language only when it is pedagogically helpful.

1. **[A-SSE, S-IC]** The chance of success in a large number of independent trials, each with a small probability of success, such as these examples about animals "predicting" results of sporting events. Data science benefits include learning to be thoughtful about assumptions of independence, and identifying factors to consider when assessing whether an outcome is surprising.

2. **[F-IF, F-LE]** Exponential approximation of the chance of at least one success in *n* independent trials with success probability *1/n* on each single trial, when *n* is large. Such questions can be motivated in many different contexts including resampling. In Algebra II, not all schools introduce *e* as the base of the natural logarithm. In schools that do not, the example should start with students graphing the logarithm function (with different bases) in a small interval around *x=1*, and observing the slope of the tangent line at *x=1* (no calculus involved). Then *e* can simply be defined as the base that makes the slope equal to 1. Mathematical benefits include a clear visual sense of the graph of

the natural logarithm function near 1, as well as quick "look, no calculator" approximations such as *ln*(1.01) is approximately 0.01 and *ln*(0.99) is approximately –0.01. Data science benefits include an introduction to a powerful and frequently used technique for approximating probabilities.

3. **[A-APR, A-SSE, S-IC]** The binomial distribution. This does not require prior knowledge of the binomial theorem: the combinatorial term can simply be introduced as the relevant count. Data science benefits include not relying on simulated data to estimate binomial probabilities without any sense of how good the estimates are. These errors are particularly troublesome when the probabilities being estimated are small, such as small *p*-values or the chance of exactly 50% heads in a large number of tosses of a fair coin. Indeed, the binomial distribution can be used to quantify the error in probability estimates based on simulation.

4. Clarity about deterministic (that is, non-random) functions and their approximations.
- The normal approximation to the binomial: a mathematical property of a class of deterministic functions
- The normal or other distributional approximation to an empirical distribution: an idealized mathematical function used for modeling observed data
- An empirical approximation based on simulated data: a random approximation to a fixed quantity, that can change with each simulation and also depends on the chosen number of repetitions in the simulation

In particular, this includes understanding that the sampling distribution of a statistic is a deterministic object based on all possible samples, and that simulation produces an empirical approximation to this based on only the random subset of samples generated in that simulation. Mathematics and data science benefits are the same: precision and clarity.

5. **[A-REI, S-ID]** Bounds. If a forecast provides the chance of rain on each of the next three days, what can be said about the chance that it rains on at least one of those days? What about the chance that it doesn't rain on any of those days? If you know that the average age in a population is 40 years, what can you say about the percent of people who are more than 80 years old? How would your response change if you also knew that the standard deviation of the ages is 5 years? All of this can be done algebraically and simply, without summation notation or laborious calculation. Mathematics benefits include increased fluency with manipulating inequalities, for example when multiplying through by –1 or when inequalities involve absolute values. Data science benefits include understanding the differences between bounds, approximations, and exact values; avoiding unjustified assumptions of independence or

normality; and understanding that simple bounds can provide sufficient information for making decisions.

6. **[F-IA, F-BF, F-LE]** Using simplified versions of Tukey's ladder of powers and Bulge diagram for transformations that make distributions look more bell-shaped or scatter diagrams more linear. The mathematics benefit is to gain familiarity with the shapes of graphs of functions and their inverses—powers and roots, exponential and log functions—without the need for complicated calculations. Data science benefits include learning a commonly used method for visualization and modeling.

These examples were chosen to give a sense of the wide variety of ways in which the lower-level sequence can be utilized in a fourth year data science course. Content developers will recognize that this selection represents only a small subset of what is possible under the guiding principles listed above.

Appendix C: Preparation for Abstract Mathematical Problems, Calculus, and Beyond

We believe that a new UC Academic Senate workgroup should be put together with a longer tenure and the goal of conducting a deeper investigation of how mathematics education at the high school level can be improved to increase the ability of students to solve abstract mathematical problems and gain a deep understanding of concepts needed for calculus and beyond, and whether changes to Senate Regulations requiring full Senate review may be needed to accomplish this.

The time given to our workgroup was insufficient to tackle a problem of this magnitude. The advocates for the formation of the new Senate workgroup believe that it should consist of: (1) STEM UC faculty from each UC campus who have an understanding of what mathematics preparation is appropriate for STEM majors; (2) Non-STEM faculty from each UC campus who have an understanding of what mathematics preparation is appropriate for non-STEM majors; and (3) Faculty with expertise in methods for enhancing the quality of K-12 mathematics education and the constraints that California high schools face.

ACW members have the expertise required for Group (1) above. We recommend some overlap between the new workgroup and the ACW, whose members have already engaged in closely related work.